Pre-Calculus CP 1 – Section 3.5 Notes **Name:** Key

MORE Application of Exponential and Logarithmic Functions

Sometimes you will need to develop the equation on your own, and then solve for the variable inside that equation. The most common problems will have to do with investments and growth/decay.

$A = P\left(1+\frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$

INVESTING!

1) How long will it take for \$1000 (P) to grow into \$1500 (A) if it earns 8% ($r=.08$) annual interest, compounded monthly?

$n=12$

$1500 = 1000\left(1+\frac{.08}{12}\right)^{12t}$

$\frac{3}{2} = \frac{1500}{1000} = (1.0067)^{12t}$

$\log_{1.0067}\frac{3}{2} = 12t$

$\frac{\ln 3/2}{\ln 1.0067} = 12t$

$t = \frac{\ln 3/2}{\ln 1.0067}\cdot\frac{1}{12}$

$= 5.06$ yrs

2) How long will it take for an investment to double if it earns 6.5% annual interest, compounded daily?

$n=365$ $A=2, P=1$ $r=.065$

$2 = 1\left(1+\frac{.065}{365}\right)^{365t}$

$2 = (1.000178)^{365t}$

switcheroo!

$\log_{1.000178} 2 = 365t$

change of base

$\frac{\ln 2}{\ln 1.000178}\cdot\frac{1}{365} = t$

$t = 10.67$ yrs

MORE Application of Exponential and Logarithmic Functions

POPULATION GROWTH and DECAY!

$A = Pe^{rt}$

3) A population of fruit flies is increasing according to the law of exponential growth. After 2 days, there are 100 flies, and after 4 days, there are 300 flies. How many flies will there be after 5 days?

t		
	2	100 ← initial day
2	4	300
3	5	?

$300 = 100e^{2r}$

$3 = e^{2r}$

$\ln 3 = 2r$

$r = \frac{\ln 3}{2}$

$A = 100e^{\left(\frac{\ln 3}{2}\right)3}$

$= 519.615$

≈ 520 flies

4) To estimate the age of dead organic material, scientists use the following formula, which denotes the ratio of carbon 14 to carbon 12 present at any time t (in years)

$$R = \frac{1}{10^{12}} e^{-t/8223}$$

Estimate the **age** of a newly discovered fossil in which the ratio of carbon 14 to carbon 12 is $R = \frac{1}{10^{13}}$.

$\frac{1}{10^{13}} = \frac{1}{10^{12}} e^{-t/8223}$

$\frac{10^{12}}{10^{13}} = \frac{1}{10} = e^{-t/8223}$

switcheroo!

$\ln \frac{1}{10} = \frac{-t}{8223}$

$t = -8223 \ln \frac{1}{10} \approx 18{,}934.16$ yrs

MORE Application of Exponential and Logarithmic Functions

5) At Dover-Sherborn High School, with a population of 600 students, one student returned from February vacation from Italy with the highly contagious corona virus (this is not a true story, nor an accurate model, but…). The spread of the virus is modeled by:

$$y = \frac{600}{1+599e^{-0.8t}},\ t \geq 0$$

where y is the total number of students infected after t days. The high school will close when 40% or more of the students are infected.

$t=5$

a) How many students are infected after 5 days?

$$y = \frac{600}{1+599e^{-0.8(5)}} = 50.121 \approx \text{50 students}$$

b) After how many days will the high school have to close?

$$600 \times .4 = 240$$

$$240 = \frac{600}{1+599e^{-0.8t}}$$

$$1+599e^{-0.8t} = \frac{600}{240} = \frac{5}{2}$$

$$599e^{-0.8t} = \frac{3}{2}$$

$$e^{-0.8t} = \frac{3/2}{599}$$

$$\ln\frac{3/2}{599} = -0.8t$$

$$t = \frac{\ln\frac{3/2}{599}}{-0.8} = 7.487 \approx \text{8 days}$$

MORE Application of Exponential and Logarithmic Functions

EARTHQUAKES!

6) On the Richter scale, the magnitude R of an earthquake of intensity I is given by

$$R = \log \frac{I}{I_0}$$

where $I_0 = 1$ is the minimum intensity used for comparison.

Find the intensities (measures of wave energies) per unit of area for each earthquake:

a) Northern Sumatra in 2004: R = 9.0

$9 = \log_{10} \frac{I}{1}$

$I = 10^9 = 1,000,000,000$ wave energies

b) Southeastern Alaska in 2004: R = 6.8

$6.8 = \log_{10} I$

$I = 10^{6.8} = 6,309,573.445$ wave energies

Homework: p. 264, #7-13 odd, 37, 41, 45, 49